

Q34

$$\int \sqrt{e^x - 1} dx$$

Let $e^x = \sec^2 u$

$$\sqrt{e^x - 1} = \sqrt{\sec^2 u - 1}$$
$$\sqrt{e^x - 1} = \tan u, \text{ where } 0 < u < \frac{\pi}{2}$$
$$d e^x = d \sec^2 u$$
$$e^x dx = 2 \sec u (\sec u \tan u) du$$
$$= 2 \sec^2 u \tan u du$$
$$= 2e^x \tan u du$$
$$dx = 2 \tan u du$$
$$\int \sqrt{e^x - 1} dx$$
$$= 2 \int (\tan u) \tan u du$$
$$= 2 \int \tan^2 u du$$
$$= 2(\tan u - u) + C$$
$$= 2\sqrt{e^x - 1} - 2 \sec^{-1} \sqrt{e^x} + C$$